Calculus 2 Exam#3 (livestream solution)

(Q1.) $\sum_{n=1}^{\infty} 3\left(\frac{-1}{4}\right)^{n-1} = ?$

(A) ∞

(B) $\frac{12}{5}$

(C) $\frac{3}{4}$

(D) $\frac{2}{5}$

(E) $\frac{4}{3}$

(Q2.) Which of the following series is **absolute convergent**?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\tan^{-1}(n)}$

(B) $\sum_{n=1}^{\infty} (-1)^n$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(E) None of the above

(Q3.) Pikachu needs to find the **radius of convergence** for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 3^n}(x-2)^n$. Help him to out!

(A) $R = 3$

(B) $R = 2$

(C) $R = \frac{2}{3}$

(D) $R = \frac{3}{2}$

(E) $R = \infty$

(Q4.) Which of the following is an example of a_n that $\lim_{n\to\infty} a_n = 0$ but $\sum_{n=1}^{\infty} a_n$ diverges?

(A) $a_n = \frac{1}{n!}$

(B) $a_n = \sin\left(\frac{\pi}{2} n\right)$

(C) $a_n = 3^{-n}$

(D) $a_n = \tan^{-1}(n)$

(E) $a_n = \frac{1}{\sqrt{n}}$

(Q5.) Determine the **power series** for $\frac{5x^2}{4-x}$ at $a = 0$. State the **radius** & **interval** of convergence

(A) $\sum_{n=0}^{\infty} \frac{5}{4^{n+1}} x^{n+2}$, $R = \frac{1}{4}$, $I = \left(\frac{-1}{4}, \frac{1}{4}\right)$

(B) $\sum_{n=0}^{\infty} \frac{5}{4^{n}} x^{n+2}$, $R = \frac{1}{4}$, $I = \left(\frac{-1}{4}, \frac{1}{4}\right)$

(C) $\sum_{n=0}^{\infty} 5(4)^{n+1} x^{n+2}$, $R = 16$, $I = (-16, 16)$

(D) $\sum_{n=0}^{\infty} \frac{5}{4^{n+1}} x^{n+2}$, $R = 4$, $I = (-4, 4)$

(E) $\sum_{n=0}^{\infty} \frac{5(-1)^n}{4^{n}} x^{n+2}$, $R = 4$, $I = (-4, 4)$

(F) None of the above

(Q6.) $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\cdots=?$

(A) $e-1$ (B) $\frac{\ln 2}{3}$

(C) $\frac{\pi}{4}$ (D) $\ln 2$

(E) ∞

(Q7.) Determine if $\frac{2}{1}-\frac{4}{2}+\frac{8}{6}-\frac{16}{24}+\frac{32}{120}-\cdots$ converges or not. Justify your answer.

(Q8.) Integrate the followings as a **power series**. State the **radius** of convergence

(a) $\int \frac{\ln(1+3x)}{x}\,dx$

(b) $\int \sin(x^2)\,dx$

(Q9.) Determine if $\sum_{n=1}^{\infty}\cos^3\left(\frac{1}{n}\right)$ converges or not. Justify your answer.

(Q10.) (a) What is your **favorite sequence**? List out its first 5 terms.
Explain why it is your favorite.

(b) What is your **favorite convergent series**? What does it converge to?
Explain why it is your favorite.

(Q11.) Determine if $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{1+\ln n}}$ converges or not. Justify your answer.

(Q12.) Let a_n be the sequence $\frac{1}{8},\frac{1}{15},\frac{1}{24},\frac{1}{35},\cdots$

(a) Does a_n converge? If so, to what value?

(b) Does $\sum_{n=1}^{\infty}a_n$ converge? If so, to what value?

(Q13.) Determine the **3rd degree Taylor Polynomial** for $\cos x$ at $a=\frac{\pi}{4}$

Hint: use Taylor formula