Calculus

Applications of Differential Equations

(Q1.) Given $\frac{dA}{dt} = 0.0135A - m$ and $A(0) = 4300$. Find m so that $A(18) = 0$

Note: this will solve the problem "if Pikachu wants to pay off his $4300 debt at 16.25% APR in 18 months, then what should his monthly payment be?"

(Q2.) Police arrive at the scene of a murder at 12 am. They immediately take and record the body's temperature, which is 90 ^{O}F, and thoroughly inspect the area. By the time they finish the inspection, it is 1:30 am. They again take the temperature of the body, which has dropped to 87 ^{O}F. Assume the temperature at the crime scene has remained steady at 82 ^{O}F because of the central heater. Determine the time of death based on Newton's Law of Cooling (The rate at which the temperature of a body changes is proportional to the difference between the temperature of the body and the temperature of the surrounding).

(Q3.) A glass of cold beer initially at 36^{O}F warms up to 40^{O}F in 10 minutes while sitting in an air conditioned room of temperature of 70^{O}F. Using Newton's Law of Warming, determine the temperature of the beer after another 10 minutes.

(Q4.) Ice forms on a lake at a rate inversely proportional to the thickness of the ice (the thicker the ice, the slower the ice forms on the top until it stops forming). At t = 0th day, the ice is 2 inches thick and at t = 2nd day, the ice is 3 inches thick. How thick will the ice be at t = 4th day?

(Q5.) Solve the logistic differential equation: $\frac{dP}{dt} = KP\left(1 - \frac{P}{M}\right)$ and $P(0) = P_o$

Note: M is the carrying capacity.

(Q6.) Biologists stocked a lake with 400 fish and estimated the carrying capacity (the maximal population for the fish of that species in that lake) to be 10,000. The number of fish tripled in the first year. (a) Assuming that the size of the fish population satisfies the logistic equation, find an expression for the size of the population after t years. (b) How long will it take for the population to increase to 5000?

Note: You can use the result from (Q5.)

(Q7.) Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students (the infected student is one of the 1000 students). If it is assume that the rate at which the virus spreads is proportional not only to the number A of infected students but also to the number of students not infected, determine the number of infected students after 6 days if it is further observed that after 4 days $A(4) = 50$.

Note: You can use the result from (Q5.)

Reference: Sect 9.3, 9.4, Single Variable Calculus by James Stewart, 8th edition

(Q8.) A tank contains 60 gallons of a solution composed of 85% water and 15% alcohol. A second solution containing half water and half alcohol is added to the tank at the rate of 4 gallons per minutes. At the same time, the tank is being drained at the same rate. Assuming that the solution is stirred constantly, how much alcohol will be in the tank after 10 minutes?

(Q9.) A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 25 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after half an hour?

(Q10.) A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 7% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. (a) What is the amount of alcohol after an hour? (b) What is the percentage of alcohol after an hour?

Reference: Sect 9.3, 9.4, Single Variable Calculus by James Stewart, 8th edition