Calculus
Introduction to Differential Equations

(Q1.) Verify that $y = \frac{2}{3}e^t + e^{-2t}$ is a solution of $y' + 2y = 2e^t$

(Q2.) Verify that $y = -\frac{1}{2}t\cos t$ is a solution to $\dfrac{d^2 y}{dt^2} + y = \sin t$

(Q3.) Verify that $y = \sin(\ln x)$ is a solution to $x^2 y'' + xy' + y = 0$

(Q4.) (a) Verify that $y = Ce^x - 3x - 3$ is the general solution to $\dfrac{dy}{dx} = 3x + y$

   (b) Solve for C if $y(0) = -1$

(Q5.) (a) Verify that $y = \sin x + C\cos x$ is the general solution to $\dfrac{dy}{dx} + (\tan x)y = \sec x$

   (b) Solve for C if $y(\pi) = 2$

(Q6.) (a) Verify that $y = C_1 e^{-4x} + C_2 e^{2x}$ is the general solution to $y'' + 2y' - 8y = 0$

   (b) Solve for C_1 and C_2 if $y(0) = 2$ and $y'(0) = 3$

(Q7.) (a) Verify that $y = C_1 \sin(2x) + C_2 \cos(2x) + 2$ is the general solution to $y'' + 4y = 8$

   (b) Solve for C_1 and C_2 if $y(0) = 5$ and $y'(0) = -1$

(Q8.) For what values of r will the function $y = e^{rx}$ satisfy $2y'' + y' - y = 0$?

(Q9.) For what values of r will the function $y = \cos(rx)$ satisfy $4y'' + 25y = 0$?

(Q10.) For what values of r will the function $y = x^r$ satisfy $x^2 y'' + 6xy' + 6y = 0$?

(Q11.) (a) Is $\dfrac{d^2 y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$ always true, sometimes true or never true?

   (b) Is $y = -\ln(2 - x)$ is a solution to $\dfrac{d^2 y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$?

   (c) Is $y = -\sqrt{2 - x}$ is a solution to $\dfrac{d^2 y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$?

(Q12.) Consider a population modeled by the differential equation $\dfrac{dP}{dt} = 0.13P\left(1 - \dfrac{P}{20500}\right)$
   (a) For what values of P is the population increasing?
   (b) For what values of P is the population decreasing?
   (c) What are the equilibrium solutions?

(Q13.) Consider an amount modeled by the differential equation $\dfrac{dA}{dt} = 0.002A^2 - A$
   (a) For what values of A is the amount increasing?
   (b) For what values of A is the amount decreasing?
   (c) What are the equilibrium solutions?