Calculus 2 Exam#3, (sequence & series)

(Q1.) $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\cdots=?$

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\ln 2}{2}$

(D) $\ln 2$ (E) ∞

(Q2.) Which of the following series **converges absolutely**?

(A) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n}}$ (B) $\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n}$ (C) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^3+1}$

(D) $\sum_{n=1}^{\infty}(-1)^n$ (E) $\sum_{n=1}^{\infty}\frac{(-1)^n 3^n}{2^n}$

(Q3.) If possible, evaluate $\sum_{n=1}^{\infty}\left(e^{\frac{1}{\sqrt{n}}}-e^{\frac{1}{\sqrt{n+1}}}\right)$

(A) e (B) $e-1$ (C) $e+e^2$

(D) e^2-e (E) this series diverges

(Q4.) Which of the following infinite series diverges by the **Test for Divergence**?

(A) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}\ln(n)}$ (B) $\sum_{n=1}^{\infty}\frac{\tan^{-1}(n)}{n^2}$ (C) $\sum_{n=1}^{\infty}\frac{n^2+2n}{n^3+4n+9}$

(D) $\sum_{n=1}^{\infty}\cos\left(\frac{1}{n}\right)$ (E) $\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$

(Q5.) Pikachu needs to find the **radius of convergence** for the power series $\sum_{n=1}^{\infty}2^n(x-3)^n$.
Help him to out!

(A) $R=3$ (B) $R=2$ (C) $R=\frac{1}{2}$

(D) $R=4$ (E) $R=\frac{1}{4}$

(Q6.) Consider a sequence defined recursively by $a_1=5$, $a_n=8-a_{n-1}$ for $n\geq 2$. Which of the following statement about a_n is **true**?

(A) a_n diverges (B) a_n converges to 3 (C) a_n converges to 5

(D) a_n is increasing (E) a_n is decreasing

(Q7.) Determine the **first four nonzero terms** of the power series for

$$\ln x \text{ at } a = 2$$

(Q8.) Integrate the followings as a **power series**. State the **radius** of convergence

(a) $\int e^{-x^2}\,dx$

(b) $\int \frac{1}{1+8x^3}\,dx$

(Q9.) Determine if $\sum_{n=1}^{\infty} \sin^3\left(\frac{1}{n}\right)$ converges or not. Justify your answer.

(Q10.) Bench press 225 pounds, 5 reps.

(Q11.) Determine if $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n!}}$ converges or not. Justify your answer.

(Q12.) Let $a_n = 2\left(\frac{-3}{4}\right)^n$

(a) Does a_n converge? If so, to what value?

(b) Does $\sum_{n=1}^{\infty} a_n$ converge? If so, to what value?

(Q13.) Determine if $\sum_{n=1}^{\infty} \frac{n^2+2n+4}{\sqrt{n^5+8n^2-2}}$ converges or not. Justify your answer.

(Q14.) Give an example for each part below

(a) a_n so that $\sum_{n=1}^{\infty} a_n = 2$

(b) a_n and b_n so that both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge but $\sum_{n=1}^{\infty} (a_n b_n)$ converges

(c) a_n so that $a_n \neq 0$ for all n but $\sum_{n=1}^{\infty} a_n = 0$ *not on the actual exam*

(d) a_n and b_n so that $a_n \neq b_n$ but $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} b_n$ *not on the actual exam*