

Inverse Laplace Marathon

(great for differential equations students)

Video: <https://youtu.be/Gx6yQcmLuH8>

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$$(Q1.) \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$$

$$(Q2.) \mathcal{L}^{-1}\left\{\frac{1}{6s+3}\right\}$$

$$(Q3.) \mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\}$$

$$(Q4.) \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s}\right\}$$

$$(Q5.) \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2}\right\}$$

$$(Q6.) \mathcal{L}^{-1}\left\{\frac{se^{-\frac{\pi}{2}s}}{s^2+1}\right\}$$

$$(Q7.) \mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\}$$

$$(Q8.) \mathcal{L}^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}$$

$$(Q9.) \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^5}\right\}$$

$$(Q10.) \mathcal{L}^{-1}\left\{\frac{1}{\sqrt{s}} + \frac{1}{\sqrt{e^s}}\right\}$$

$$(Q11.) \mathcal{L}^{-1}\left\{\frac{s+8}{s^2+4s+13}\right\}$$

$$(Q12.) \mathcal{L}^{-1}\left\{\frac{1}{s^4+5s^2+4}\right\}$$

$$(Q13.) \mathcal{L}^{-1}\left\{\frac{1}{s^4 e^{10s}}\right\}$$

$$(Q14.) \mathcal{L}^{-1}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\}$$

$$(Q15.) \mathcal{L}^{-1}\left\{\ln\left(\frac{s^2+9}{s^2+1}\right)\right\}$$

$$(Q16.) \mathcal{L}^{-1}\left\{\frac{1}{s^4-16}\right\}$$

$$(Q17.) \mathcal{L}^{-1}\left\{\frac{s^3}{(s^4-16)^2}\right\}$$

$$(Q18.) \mathcal{L}^{-1}\left\{\frac{1}{s^4 + 4s^2 + 4}\right\}$$

$$(Q19.) y' + 2y = \sin(3t), y(0) = 0$$

$$(Q20.) y'' - 4y = 3t^2, y(0) = 2, y'(0) = -4$$

$$(Q21.) y'' - 2y' - 15y = 0, y(0) = -2, y'(0) = 1$$

$$(Q22.) y'' + 16y = \cos(4t), y(0) = 0, y'(0) = 1$$

$$(Q23.) y'' + y = \mathcal{U}(t - 3), y(0) = 0, y'(0) = 1$$

$$(Q24.) y'' - 4y' + 5y = 0, y(0) = 1, y'(0) = 2$$

$$\begin{aligned}
(\text{A1.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} &= \frac{t^3}{6} \\
(\text{A2.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{6s+3}\right\} &= \frac{1}{6}e^{-\frac{1}{2}t} \\
(\text{A3.}) \quad \mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\} &= \cos(\sqrt{2}t) + \frac{1}{\sqrt{2}}\sin(\sqrt{2}t) \\
(\text{A4.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s}\right\} &= \frac{1}{2} - \frac{e^{-2t}}{2} \\
(\text{A5.}) \quad \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2}\right\} &= e^{-2t} - 2te^{-2t} \\
(\text{A6.}) \quad \mathcal{L}^{-1}\left\{\frac{se^{-\frac{\pi}{2}s}}{s^2+1}\right\} &= \cos\left(t - \frac{\pi}{2}\right)\mathcal{U}\left(t - \frac{\pi}{2}\right) = \sin t \mathcal{U}\left(t - \frac{\pi}{2}\right) \\
(\text{A7.}) \quad \mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} &= e^{-t}\cos t - e^{-t}\sin t \\
(\text{A8.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\} &= \frac{1}{2}t^2 + \cos t - 1 \\
(\text{A9.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^5}\right\} &= \frac{1}{24}e^{-2t}t^4 \\
(\text{A10.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{\sqrt{s}} + \frac{1}{\sqrt{e^s}}\right\} &= \frac{1}{\sqrt{\pi t}} + \delta\left(t - \frac{1}{2}\right) \\
(\text{A11.}) \quad \mathcal{L}^{-1}\left\{\frac{s+8}{s^2+4s+13}\right\} &= e^{-2t}\cos(3t) + 2e^{-2t}\sin(3t) \\
(\text{A12.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{s^4+5s^2+4}\right\} &= \frac{1}{3}\sin t - \frac{1}{6}\sin(2t) \\
(\text{A13.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{s^4e^{10s}}\right\} &= \frac{1}{6}(t-10)^3\mathcal{U}(t-10) \\
(\text{A14.}) \quad \mathcal{L}^{-1}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\} &= \frac{\sin t}{t} \\
(\text{A15.}) \quad \mathcal{L}^{-1}\left\{\ln\left(\frac{s^2+9}{s^2+1}\right)\right\} &= \frac{-2\cos(3t) + 2\cos t}{t} \\
(\text{A16.}) \quad \mathcal{L}^{-1}\left\{\frac{1}{s^4-16}\right\} &= -\frac{1}{32}e^{-2t} + \frac{1}{32}e^{2t} - \frac{1}{16}\sin(2t) \\
(\text{A17.}) \quad \mathcal{L}^{-1}\left\{\frac{s^3}{(s^4-16)^2}\right\} &= \frac{t}{4}\left(-\frac{1}{32}e^{-2t} + \frac{1}{32}e^{2t} - \frac{1}{16}\sin(2t)\right)
\end{aligned}$$

$$(A18.) \mathcal{L}^{-1}\left\{\frac{1}{s^4 + 4s^2 + 4}\right\} = \frac{\sqrt{2}}{8}\sin(\sqrt{2}t) - \frac{1}{4}t\cos(\sqrt{2}t)$$

$$(A19.) y' + 2y = \sin(3t), y(0) = 0 \quad \underline{\text{Ans:}} y(t) = \frac{3}{13}e^{-2t} - \frac{3}{13}\cos(3t) + \frac{2}{13}\sin(3t)$$

$$(A20.) y'' - 4y = 3t^2, y(0) = 2, y'(0) = -4 \quad \underline{\text{Ans:}} y(t) = \frac{3}{16}e^{2t} + \frac{35}{16}e^{-2t} - \frac{3}{4}t^2 - \frac{3}{8}$$

$$(A21.) y'' - 2y' - 15y = 0, y(0) = -2, y'(0) = 1 \quad \underline{\text{Ans:}} y(t) = \frac{-5}{8}e^{5t} - \frac{11}{8}e^{-3t}$$

$$(A22.) y'' + 16y = \cos(4t), y(0) = 0, y'(0) = 1 \quad \underline{\text{Ans:}} y(t) = \frac{1}{4}\sin(4t) + \frac{1}{8}t\sin(4t)$$

$$(A23.) y'' + y = \mathcal{U}(t-3), y(0) = 0, y'(0) = 1 \quad \underline{\text{Ans:}} y(t) = \sin t + \mathcal{U}(t-3)(1 - \cos(t-3))$$

$$(A24.) y'' - 4y' + 5y = 0, y(0) = 1, y'(0) = 2 \quad \underline{\text{Ans:}} y(t) = e^{2t}\cos t$$