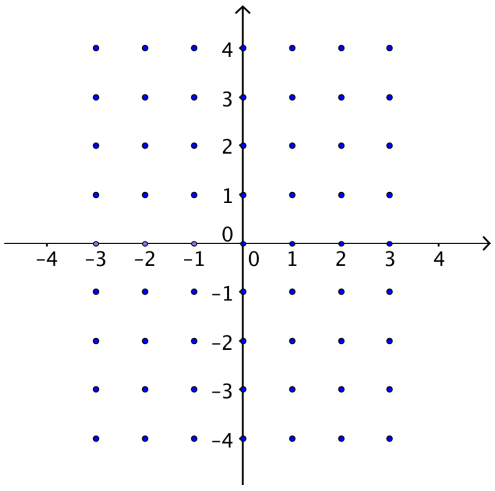
# Calculus

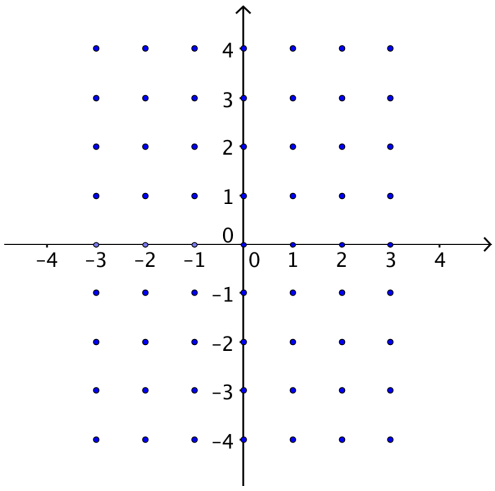
## Slope Field & Euler's Method

(Ex1.) Sketch the slope field for $\frac{dy}{dx} = 4 - y^2$

(a) Sketch a **solution curve** to the differential equation, with a given point (0, –1)

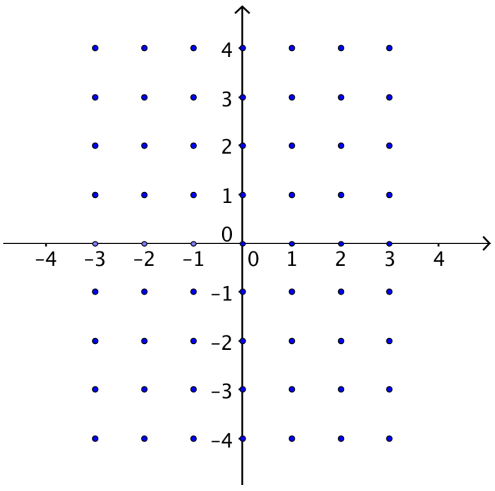
(b) Sketch a **solution curve** to the differential equation, with a given point (0, 3)




(Ex2.) Sketch the slope field for $\frac{dy}{dx} = xy$

Sketch a **solution curve** to the differential equation, with a given point (1, 2)

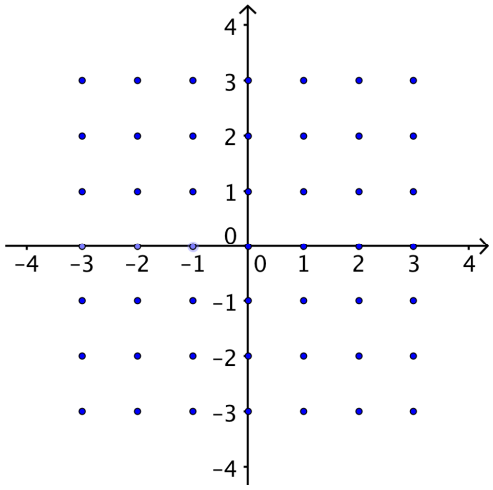


(Q1.) Consider $\frac{dy}{dx} = 3x + y$

(a) Sketch a **slope field**

(b) Sketch a **solution curve** to the differential equation, with a given point $(0, -1)$

(c) Let $y(x)$ be the solution to the differential equation and $y(0) = -1$.
Use **Euler's Method**, with step size $0.04$, to estimate $y(0.2)$



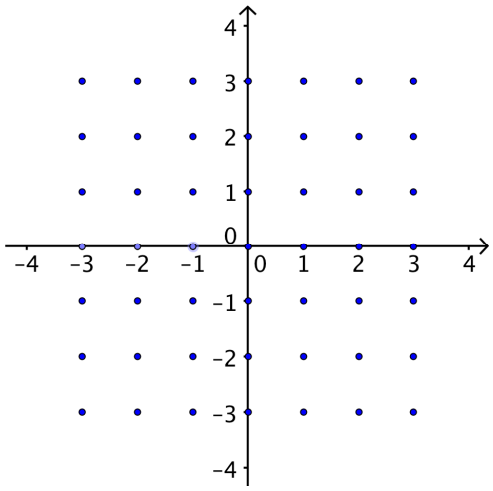
(Q2.) Consider $\frac{dy}{dx} = \frac{x}{y}$

(a) Sketch a **slope field**

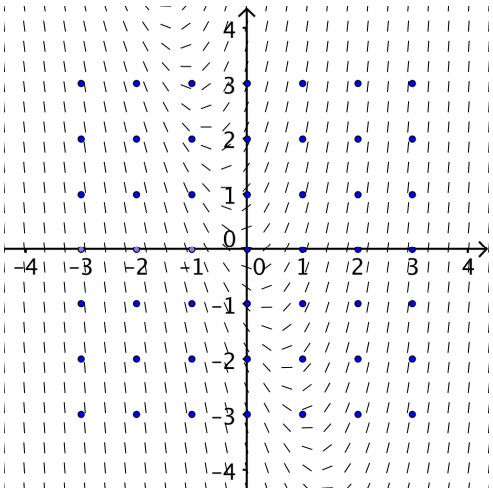
(b) Sketch a **solution curve** to the differential equation, with $y(2) = \sqrt{3}$

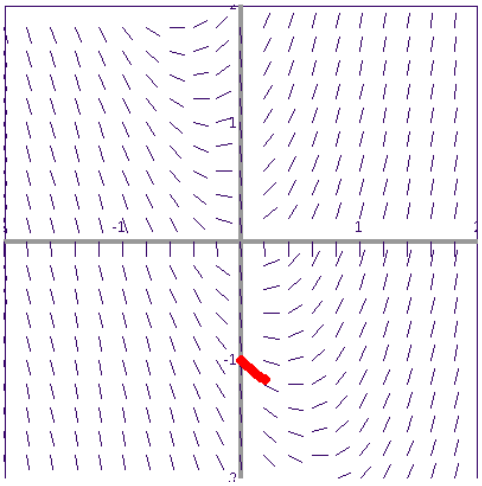
(c) Let $y(x)$ be the solution to the differential equation and $y(2) = \sqrt{3}$.
Use **Euler's Method**, with step size $\frac{1}{3}$, to estimate $y(3)$

(d) Solve the differential equation, with $y(2) = \sqrt{3}$. Evaluate $y(3)$.



(Q1.) $y = 2e^{x} - 3x - 3$

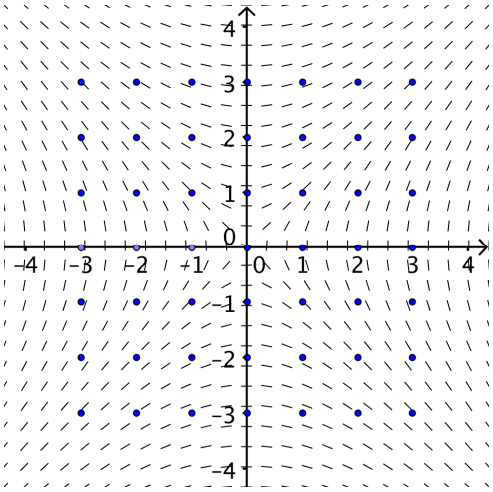


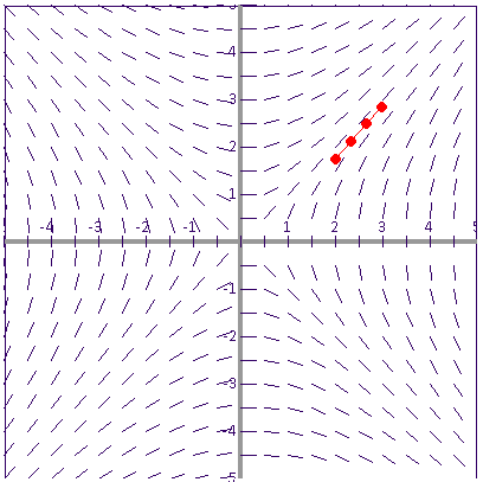


**Equation : 3x+y**

1) (0.0000, -1.0000)
2) (0.0400, -1.0400)
3) (0.0800, -1.0768)
4) (0.1200, -1.1103)
5) (0.1600, -1.1403)
6) (0.2000, -1.1667)

(Q2.) $y = \sqrt{x^2 - 1}$





**Equation : x/y**

1) (2.0000, 1.7320)
2) (2.3333, 2.1169)
3) (2.6667, 2.4843)
4) (3.0000, 2.8421)

**Euler's Method**: Given $(x_0, y_0)$, $\frac{dy}{dx} = F(x,y)$, step size = $h$

$$x_n = x_{n-1} + h \qquad y_n = y_{n-1} + h \cdot F(x_{n-1}, y_{n-1})$$