Calculus
Sequences

For Q1 to Q11, (a) Find an explicit formula for a_n (b) determine if a_n is increasing, decreasing or neither (you may use a graph) (c) determine if a_n converge.

(Q1.) $\frac{3}{7}, \frac{6}{14}, \frac{9}{28}, \frac{12}{56}, \frac{15}{112}, \ldots$

(Q2.) $\frac{1}{3}, \frac{1}{8}, \frac{1}{15}, \frac{1}{24}, \frac{1}{35}, \ldots$

(Q3.) 5, 7, 5, 7, 5, 7, ...

(Q4.) $\frac{1}{2!}, \frac{1}{4!}, \frac{1}{6!}, \frac{1}{8!}, \frac{1}{10!}, \ldots$

(Q5.) $\sin(1), 2\sin(\frac{1}{2}), 3\sin(\frac{1}{3}), 4\sin(\frac{1}{4}), 5\sin(\frac{1}{5}), \ldots$

(Q6.) $\cos(1), \cos(\frac{1}{2}), \cos(\frac{1}{3}), \cos(\frac{1}{4}), \cos(\frac{1}{5}), \ldots$

(Q7.) $\frac{1}{\tan^{-1}(1)}, \frac{1}{\tan^{-1}(2)}, \frac{1}{\tan^{-1}(3)}, \frac{1}{\tan^{-1}(4)}, \frac{1}{\tan^{-1}(5)}, \ldots$

(Q8.) $\frac{\sqrt{2}}{\ln 2}, \frac{\sqrt{3}}{\ln 3}, \frac{\sqrt{4}}{\ln 4}, \frac{\sqrt{5}}{\ln 5}, \frac{\sqrt{6}}{\ln 6} \ldots$

(Q9.) $\frac{1}{3}, \frac{1}{5}, \frac{1}{9}, \frac{1}{17}, \frac{1}{33} \ldots$

(Q10.) $\frac{1}{1}, \frac{-1}{8}, \frac{1}{27}, \frac{-1}{64}, \frac{1}{125}, \ldots$

(Q11.) $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}, \sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}, \ldots$

(Q12.) $\frac{d^{125}}{dx^{125}}(\cos(2x)) = ?$

(Q13.) Find a differentiable function so that

$$f(1)=0,\ f(2)=0,\ f(3)=0,\ f(4)=1,\ f(5)=0,\ f(6)=0,\ f(7)=0,\ f(8)=1,\ \ldots$$

(Q14.) Give an example of $f(x)$, where $f(n)=a_n$, such that $\lim_{n\to\infty} a_n = L$ but $\lim_{x\to\infty} f(x) \neq L$

(Q15.) Find an explicit formula for the sequence $-2, 3, 12, 25, 42, \ldots$

hint: this is a quadratic sequence and has the general form $a_n = An^2 + Bn + C$

Reference: Sect 11.1, Single Variable Calculus by James Stewart, 8th edition

(Q16.) Consider $\begin{cases} a_1 = 1 \\ a_n = 1 + \dfrac{1}{1 + a_{n-1}}, \text{ for } n \geq 2 \end{cases}$

(a) Write out the first 8 terms. Round your answers to 5 decimal places. Notice that a_n appears to be a convergent sequence.

(b) Given a_n does converge. Evaluate $\lim_{n\to\infty} a_n$ from the recursive formula

(Q17.) Consider $\frac{3}{4}, \frac{4}{7}, \frac{5}{10}, \frac{6}{13}, \frac{7}{16}, \ldots$

(a) Find an explicit formula for a_n

(b) Given a_n converges. Evaluate $\lim_{n\to\infty} a_n$ from the explicit formula

(c) Find an recursive formula for a_n

(d) Given a_n converges. Evaluate $\lim_{n\to\infty} a_n$ from the recursive formula

Reference: Sect 11.1, Single Variable Calculus by James Stewart, 8th edition