Calculus

Alternating Series Test & Absolute Convergence

(Q1.) Give an example of a_n so that $a_n \geq 0$ and $\lim_{n\to\infty} a_n = 0$ but a_n is **NOT** decreasing.

(Q2.) Give an example of a_n so that **both** $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} |a_n|$ **converge**.

(Q3.) Give an example of a_n so that **both** $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} |a_n|$ **diverges**.

(Q4.) Give an example of a_n and b_n so that both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge but $\sum_{n=1}^{\infty} (a_n b_n)$ converges.

(Q5.) Give an example of a_n and b_n so that both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge but $\sum_{n=1}^{\infty} (a_n b_n)$ diverges.

(Q6.) to (Q14.) Determine whether the series converges absolutely, conditionally or diverge?

(Q6.) $\frac{1}{\ln 2} - \frac{1}{\ln 4} + \frac{1}{\ln 6} - \frac{1}{\ln 8} + \frac{1}{\ln 10} - \cdots$

(Q7.) $-\frac{1}{4} + \frac{1}{13} - \frac{1}{22} + \frac{1}{31} - \frac{1}{40} + \cdots$

(Q8.) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \cdots$

(Q9.) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

(Q10.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

(Q11.) $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{n^2}$

(Q12.) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n+1}$

(Q13.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\tan^{-1} n}$

(Q14.) $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n+3^n}$

Optional Challenges: Evaluate $\sum_{n=1}^{\infty} a_n$ where $a_1 = 400$, $a_2 = 200$ and $a_n = \frac{a_{n-1} + a_{n-2}}{2}$ for $n \geq 3$

Reference: Sect 11.5, Single Variable Calculus by James Stewart, 8th edition