

Calculus

Best Friend & Other Friends

$$\frac{1}{1-x} = 1+x+x^2+x^3+\dots = \sum_{n=0}^{\infty} x^n \quad R=1 \quad I=(-1, 1)$$

$$\frac{1}{(1-x)^2} = 1+2x+3x^2+4x^3+\dots = \sum_{n=1}^{\infty} nx^{n-1} \quad R=1 \quad I=(-1, 1)$$

$$\frac{1}{1+x} = 1-x+x^2-x^3+\dots = \sum_{n=0}^{\infty} (-1)^n x^n \quad R=1 \quad I=(-1, 1)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1} \quad R=1 \quad I=(-1, 1]$$

$$\frac{1}{1+x^2} = 1-x^2+x^4-x^6+\dots = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad R=1 \quad I=(-1, 1)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \quad R=1 \quad I=[-1, 1]$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \quad R=\infty \quad I=(-\infty, \infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \quad R=\infty \quad I=(-\infty, \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \quad R=\infty \quad I=(-\infty, \infty)$$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

Famous Sums:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots = \ln 2$$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

$$1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots = e - 1$$

$$0.9999999999999999... = 1$$

$$0.012345679012345679... = \frac{1}{81}$$