Calculus

Sequences & Their Formulas

(Q1.) $\frac{4}{9}$, $\frac{5}{11}$, $\frac{6}{13}$, $\frac{7}{15}$, $\frac{8}{17}$, _____, _____, …

(Q2.) 243, 162, 108, 72, 48, _____, _____, …

(Q3.) 1, 2, 6, 24, 120, _____, _____, …

(Q4.) 2, 6, 12, 20, 30, _____, _____, …

(Q5.) 141, 133, 125, 117, 109, _____, _____, …

(Q6.) 0, 1, 3, 7, 15, _____, _____, …

(Q7.) 1, 2, 3, 4, 5, 8, _____, _____, … (there is **NO** typo in this sequence)

(Q8.) 0, 1, 0, 1, 0, _____, _____, …

Calculus

Sequences & Series

[Explicit Formula] *The new term is based on the value of n*

Example: $a_n = \frac{n^2}{2n+1}$ First five terms: $\overbrace{\frac{1}{3}}^{a_1}, \overbrace{\frac{4}{5}}^{a_2}, \overbrace{\frac{9}{7}}^{a_3}, \overbrace{\frac{16}{9}}^{a_4}, \overbrace{\frac{25}{11}}^{a_5}$

n	1	2	3	4	5
$a_n = \frac{n^2}{2n+1}$	$\frac{(1)^2}{2(1)+1}$	$\frac{(2)^2}{2(2)+1}$	$\frac{(3)^2}{2(3)+1}$	$\frac{(4)^2}{2(4)+1}$	$\frac{(5)^2}{2(5)+1}$

[Recursive Formula] *The new term is based on the previous terms*

Example: $a_1 = -3,\ \underbrace{a_n}_{\text{new term}} = 4\ \underbrace{a_{n-1}}_{\text{previous term}} + 7$ First five terms: $\overbrace{-3}^{a_1}, \overbrace{-5}^{a_2}, \overbrace{-13}^{a_3}, \overbrace{-45}^{a_4}, \overbrace{-173}^{a_5}$

n	1	2	3	4	5
$a_n = 4a_{n-1} + 7$	given to be -3	$4(-3)+7$ $= -5$	$4(-5)+7$ $= -13$	$4(-13)+7$ $= -45$	$4(-45)+7$ $= -173$

[Series] Adding up the numbers in a sequence

Example: $\sum_{n=2}^{6}(8-2n)$

$$\sum_{n=2}^{6}(8-2n) = \overbrace{(8-2(2))}^{n=2} + \overbrace{(8-2(3))}^{n=3} + \overbrace{(8-2(4))}^{n=4} + \overbrace{(8-2(5))}^{n=5} + \overbrace{(8-2(6))}^{n=6}$$

	Arithmetic Sequence Keep adding a number: common difference d	**Geometric Sequence** Keep multiplying a number: common ratio r
n^{th} term	$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$
Sum of the first n terms	$s_n = \frac{n}{2}(a_1 + a_n)$	$s_n = \frac{a_1(1-r^n)}{1-r}$