Calculus

Arc Length & Surface Area

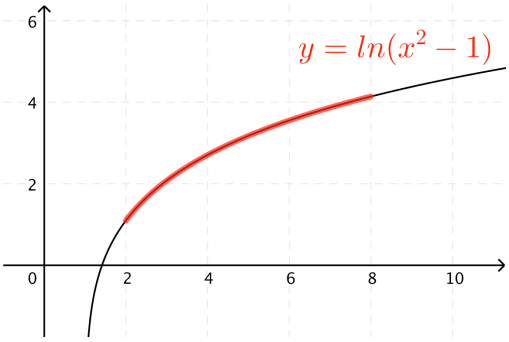
Evaluate your integrals with your graphing calculator for (Q1.) to (Q5.)

(Q1.) Consider the arc on the curve $y = \ln(x^2 - 1)$ from $x = 2$ to $x = 8$

(a) Find the **arc length**

(b) Find the **surface area** obtained by rotating this arc about the x-axis

(c) Find the **surface area** obtained by rotating this arc about the y-axis

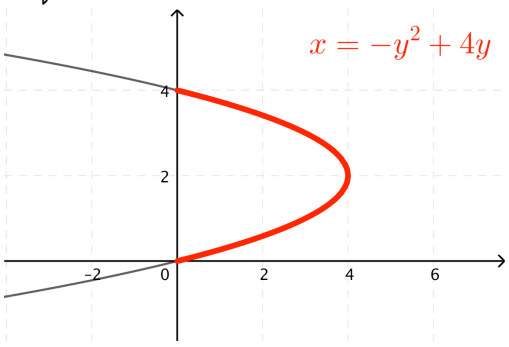


(Q2.) Consider the arc on the curve $x = -y^2 + 4y$ in the first quadrant

(a) Find the **arc length**

(b) Find the **surface area** obtained by rotating this arc about the x-axis

(c) Find the **surface area** obtained by rotating this arc about the y-axis

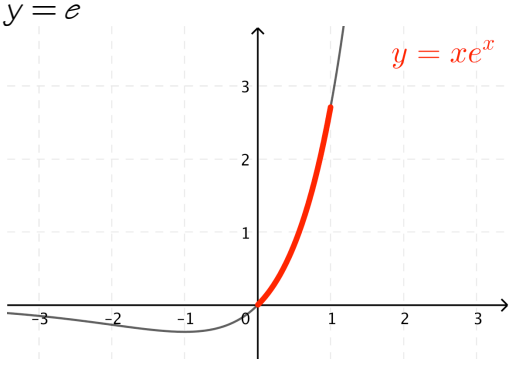


(Q3.) Consider the arc on the curve $y = xe^x$ from $y = 0$ to $y = e$

(a) Find the **arc length**

(b) Find the **surface area** obtained by rotating this arc about the x-axis

(c) Find the **surface area** obtained by rotating this arc about the y-axis

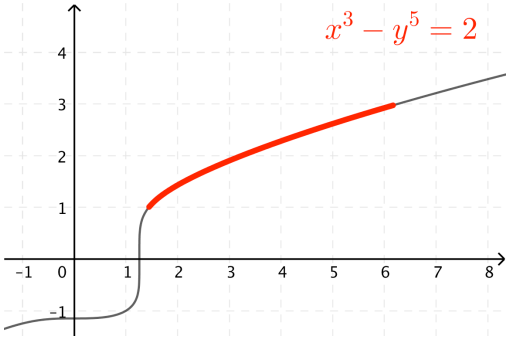


(Q4.) Consider the arc on the curve $x^3 - y^5 = 2$ from $y = 1$ to $y = 3$

(a) Find the **arc length**

(b) Find the **surface area** obtained by rotating this arc about the x-axis

(c) Find the **surface area** obtained by rotating this arc about the y-axis



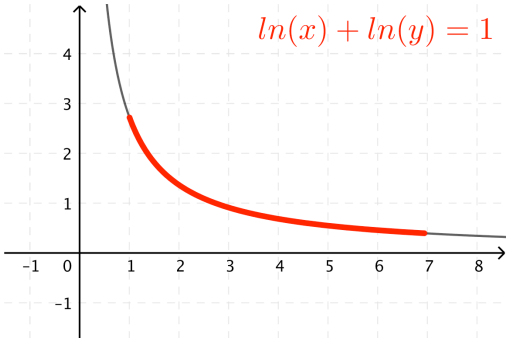
Reference: Sect 8.1, 8.2, Single Variable Calculus by James Stewart, 8th edition

(Q5.) Consider the arc on the curve $\ln x + \ln y = 1$ from $x = 1$ to $x = 7$

(a) Find the **arc length**

(b) Find the **surface area** obtained by rotating this arc about the x-axis

(c) Find the **surface area** obtained by rotating this arc about the y-axis



Do all your integrals by hand for the rest of the problems…

(Q6.) (a) Prove the **volume** of the sphere with radius r is $V = \frac{4}{3}\pi r^3$

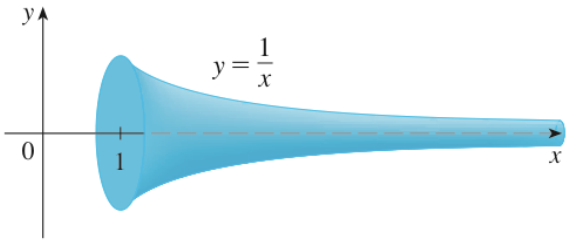
(b) Prove the **surface area** of the sphere with radius r is $S = 4\pi r^2$

Hint: Consider an equation of a circle with radius r and do a rotation

(Q7.) Consider rotating the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$ and $x = \infty$ about x-axis. This solid is called **Gabriel's Horn** (NOT because of Gabriel Iglesias)

(a) Find the **volume**

(b) Find the **surface area**



(Q8.) Just geometry here…

(a) Prove the **area of a circular sector** with radius r and angle θ is $A_{sector} = \frac{1}{2}r^2\theta$

(b) Prove the **lateral surface area of a cone** with radius r and slant height ℓ is $A_{lateral} = \pi r\ell$

(c) Prove the **lateral surface area of a frustum** (i.e. part of a cone) with radii of the bases r_1 and r_2 and slant height ℓ is $A_{band} = 2\pi r\ell$, where $r = \dfrac{r_1 + r_2}{2}$